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An experimental investigation of the composition of jet noise

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Noise intensity measurements per octave band have been carried out in the far field of a small circular nozzle, at angles 0–150° from the jet axis, with and without vortex generators. The results, interpreted in the light of recent theoretical conclusions, confirm Lighthill's original suggestion that the dominant noise radiator is the pressure–shear x–r quadrupole at all but the lowest frequencies. At very low frequencies Reynolds stress–shear x–x and x–r quadrupoles contribute comparable amounts of radiation. When the product of Strouhal number (based on nozzle diameter and jet velocity) and jet Mach number exceeds unity the geometrical acoustics approximation becomes valid, but even at such high frequencies the shear noise contribution dominates. Vortex generators, while reducing noise intensity, do not appear to modify drastically the above composition of noise radiators.

Introduction

The theory of aerodynamic noise generation by turbulent flow, particularly turbulent jets, was born with Lighthill's (1952, 1954) two classic memoirs on the subject. While there is considerable depth and detail in these papers the main conclusions may perhaps be condensed into the following three points. (i) The noise of (subsonic) jets is produced by turbulent stress fluctuations acting as acoustic quadrupoles. From this it follows at once by dimensional arguments that noise intensity is proportional to the eighth power of nozzle velocity, the celebrated 'U^8 law'. (ii) The high mean rate of strain in a jet contributes 'directly' to jet noise, apart from being 'indirectly' responsible for the existence of Reynolds stresses and the noise the fluctuations in these stresses themselves produce ('self noise'). The 'direct' or 'shear noise' contribution of the mean rate of strain is radiated as an 'x–r quadrupole', its intensity being greatest at 45° from the jet axis, except for the directional distortion noted below. (iii) The elementary noise radiators are small parcels of turbulent fluid moving at a high mean velocity. The rapid source motion distorts the directional distribution of radiation and greatly enhances forward emission. Lighthill originally proposed that, if the noise sources are convected with Mach number M_c, the directional redistribution is according to the factor $(1-M_c\cos\theta)^{-6}$, where θ is the angle from the jet axis. The total noise emission also increases (as compared with emission from stationary

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sources) leading to a theoretically predicted increase in noise power with jet velocity rather faster than the U^8 law.

While Lighthill's conclusions have been generally verified, there have been some difficulties, notably with the effect of source motion described under (iii) above. Ribner (1958) has shown that almost all the noise is radiated from the centre of the 'mixing layer' where the convection velocity is approximately 60 % of jet exit velocity. Given the corresponding convection Mach number, theory exaggerates both the forward emission and the increase in total noise power over and above the U^8 law.

One correction to the apparently too drastic convection-amplification factor $(1-M_c \cos\theta)^{-6}$ has been given by Ffowcs Williams (1960), who showed that, because the length of the noise-emitting region is defined in a *fixed* frame of reference, one step in Lighthill's original argument should be retraced, yielding the amended factor $(1-M_c \cos\theta)^{-5}$.

This amended factor was still too high and the possibility was suggested by Csanady (1966) that at least some of the noise was of sufficiently high frequency to behave according to the laws of geometrical acoustics and thus to be refracted by the velocity variations in the jet. For any such fraction of the noise the total intensity is not subject to an 'extra' increase beyond the U^8 law. This can at best offer a partial explanation of the discrepancy between theory and experiment, however, because a good deal of evidence shows that most of the noise of jets is of much too low a frequency for geometrical acoustics to hold, Lighthill's 'moving source' model (which in a sense is the opposite, low-frequency extreme) being probably a much closer approximation. Nevertheless, the question, at what frequency geometrical acoustics becomes acceptable, has never been examined experimentally; one objective of the present paper is to report such an examination.

A second correction to Lighthill's convection-amplification factor has recently been derived by Jones (1968) by a careful re-examination of the moving-source model as it applies to the shear-noise quadrupoles. His result is that, for shear noise only, the factor should be $(1-M_c \cos\theta)^{-3}$, which shows a rather less pronounced emphasis on forward emission and is generally in accord with experimental evidence on this point. The agreement with experiment suggests incidentally also that most of the jet noise is shear noise, but this requires further analysis in the light of reliable experimental data, particularly on noise intensity in the rear quadrant, $\theta = 90$–$180°$, on which previous experimental information is scant and not very reliable.

It should also be added here that a re-examination of the source terms responsible for shear noise by Csanady (1966) revealed the existence of terms in addition to Lighthill's x–r quadrupole (in particular, an x–x quadrupole). Also, these terms contain first time derivatives of stress fluctuations, whereas 'self-noise' source terms contain second time derivatives. One would therefore expect shear noise to be radiated at generally lower frequencies than self-noise. This suggests the possibility of experimentally separating shear noise from the total, at least partially.

In the light of the above it would seem to be of interest to elucidate the influence and relative importance of the various shear-noise quadrupoles and also, as remarked before, to analyze how far the geometrical acoustics approximation applies to any components of jet noise or at any rate at what frequencies refraction effects become important. An experimental investigation aimed at these questions is reported in the present paper. The method consists basically of noise intensity measurements at different angles from the jet axis in narrow frequency bands extending to angles well beyond 90°. Such data as exist in the literature (e.g. Mollö-Christensen, Kolpin & Martuccelli 1964) do not appear to be detailed or extensive enough to decide the questions raised above.

Apparatus and instrumentation

Noise measurements were made on a plot of flat grassy land two miles north of the University campus. The experimental site was free of extraneous sources of noise due to buildings, vehicular traffic and vegetation. At the experimental site very low levels of noise were recorded when the winds were light.

The photograph of the airflow-producing apparatus is shown in figure 1, plate 1. (The snow cover was absent during the experiments.) The air compressor motor unit was mounted 8 ft. below ground level in an 8 ft. × 8 ft. pit which had an acoustically well-insulated lid. Compressed air was passed to a reservoir placed outside the pit through an outlet muffler.

The nozzle with the settling chamber was placed on a horizontal axis $13\frac{1}{2}$ ft. above ground level and about 60 ft. from the air compressor pit in order to eliminate both the slight remaining noise due to the compressor unit and the effect of ground reflexion. Compressed air from the reservoir was passed through a 3 in. diameter 50 ft. long hose to the settling chamber. The settling chamber was made of aluminium, and had a diameter of 9 in. and a length of 8 ft. The settling chamber carried at one end a 2 ft. long diffuser which connected the 3 in. diameter pipe section to the 9 in. diameter section of the settling chamber. It was fitted with screens and honeycomb for the purpose of reducing turbulence. The jet nozzle was made of fibreglass and had a diameter of 1 in. The nozzle was designed for a contraction area ratio of 36:1 on the basis of data given by Smith & Wang (1946). The objective of the design was to achieve a uniform throat speed. The surface of the whole flow-producing apparatus was wound with 2 in. thick fibreglass mats in order to eliminate reflexion from the metallic surfaces.

The airflow could be varied to attain a Mach number from 0·1 to 0·7 by means of a by-pass valve near the reservoir. The air from the by-pass valve was led backwards through a long hose pipe.

All the noise-measuring instruments used were manufactured by Brüel & Kjaer. The condenser microphones used were of 1 in. and $\frac{1}{2}$ in. face diameter. The sensitivity of the 1 in. microphone was 5 mV/μbar and had a flat-frequency response within ± 1 db from 20 c/sec to 15 kc/sec. The $\frac{1}{2}$ in. condenser microphone had a flat-frequency response from 20 c/sec to 35 kc/sec within ± 1 db and had a sensitivity of 1 mV/μbar.

The 1 in. microphone with the nose cone supplied by Brüel & Kjaer was used to measure the spectrum of the low-frequency component of radiated noise. The use of a nose cone in place of a protection grid helps to reduce the wind noise and also improves the omni-directional characteristics of the microphone. The spectrum of the high-frequency component of the noise was measured by the $\frac{1}{2}$ in. condenser microphone. The microphone was set in a horizontal plane containing the jet axis mounted on a vertical pole whose height could be varied through a slider arrangement. The pole was wound with loose fibreglass near the microphones to prevent scattering by the poles. The microphone was connected to the sound level meter and filter set by an extension cable 10 ft. long supplied by Brüel & Kjaer. The attenuation introduced by the cable was $1{\cdot}4 \pm 0{\cdot}5$ db with the 1 in. microphone. The whole system was calibrated periodically, using the piston-phone supplied by the manufacturer.

The spectrum analyzer had eleven octave filter bands with central frequencies 31·5, 63, 125, 250, 500, 1000, 2000, 4000, 8000, 16,000, 31,500. Inside the filter passband the response was flat to within $\pm 0{\cdot}5$ db.

Observational procedure

Far field intensity and spectra of the noise radiated by the jet were measured at angles from 0 to 150° to the jet axis at distances 5, 10, 20 and 40 ft. from the jet nozzle. The Mach number of the issuing jet was varied from 0·4 to 0·65.

Measurements were restricted to the days when the wind velocity was light (3–6 miles/h) and the temperature 40–50 °F. The wind velocity was measured by a cup anemometer. The effect of pseudo-sound produced by the wind was noticeable up to a frequency of 250 c/s.

In some of the experiments 'vortex generators' were used to disturb the flow at the jet exit and the spectra were measured again at various angles from the jet axis. The vortex generators used consisted of eight equilateral triangles of $\frac{1}{4}$ in. side length and spacing of $\frac{1}{16}$ in. between the triangles (figure 2, plate 2). These were fitted $\frac{1}{4}$ in. downstream of the exhaust nozzle. The three sets of vortex generators tested were bent to the flow at angles 15, 30 and 40° respectively to the jet axis.

Experimental results

Useful measurements of the total noise intensity have been made at non-dimensional distances r/D of 60, 120, 240 and 480 and Mach numbers 0·5 and 0·63. Figure 3 illustrates the arrangement and the symbols used. The results were all reduced by the inverse square law to $r/D = 120$ and are shown in figure 4. The scatter of points is reasonable for acoustic measurements of this kind and indicates that the faired solid curves have an accuracy of about ± 1 db. Also, the scatter appears to be independent of distance, which shows that all measurements have indeed been carried out in the 'far field' where the inverse square law holds. The theoretical curves sketched in the figure (broken lines) will be commented upon later.

On the same basis, the noise intensity within an octave band centred at 250 c/s is shown in figure 5. At these low frequencies some difficulty was experienced with 'pseudo-sound' particularly at the larger angles where the intensity is low. The curves are nevertheless fairly well defined near the locus of peak intensity at this frequency. Figures 6–11 contain the same information for octave bands centred at successively higher frequencies, as noted.

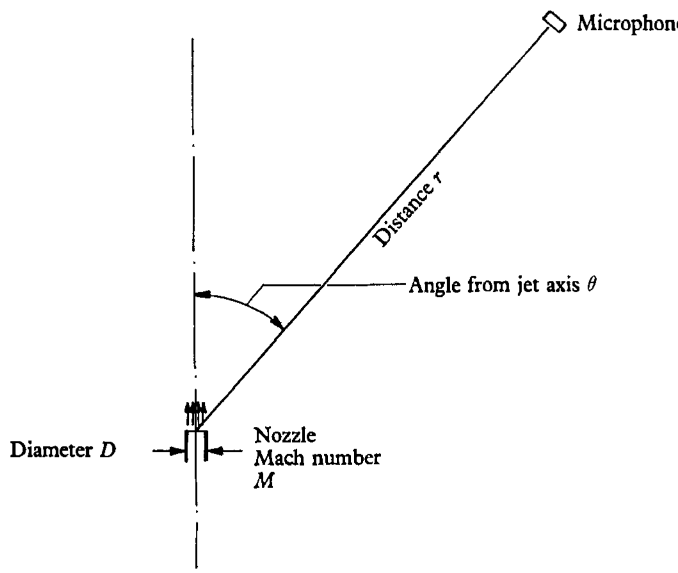

FIGURE 3. Symbols used in presenting data.

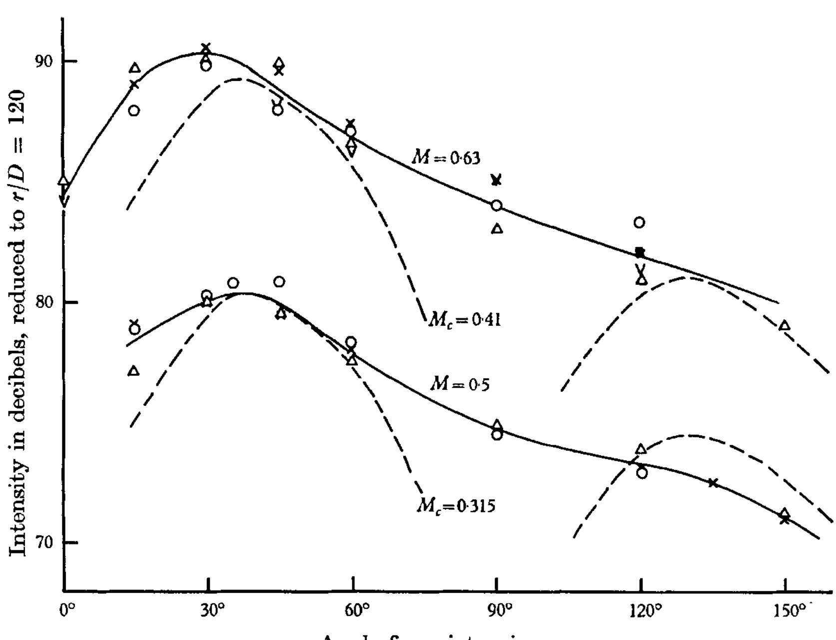

FIGURE 4. Directional distribution of total noise intensity (without vortex generators). ○, $r/D = 60$; ×, $r/D = 120$; △, $r/D = 240$; ∨, $r/D = 480$. ---, 45° shear-noise quadrupole (moving-source model) fitted to data at 45°.

While certain portions of the curves are in doubt, the angular position of the peak intensity can be determined from them with reasonable accuracy, yielding the result shown in figure 12. The fact that low and high frequencies peak at different locations has been known for some time; this graph shows the details.

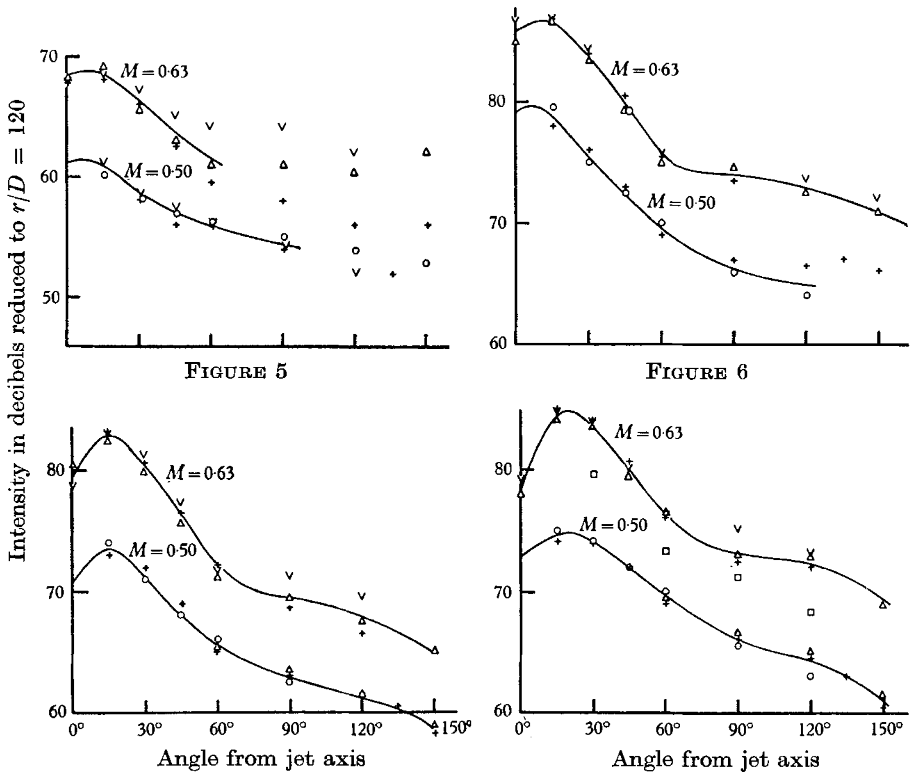

FIGURE 7 FIGURE 8

FIGURES 5–11. Directional distribution of noise intensity per octave band, central frequency: (5) 250 c/s, (6) 500 c/s, (7) 1000 c/s, (8) 2000 c/s, (9) 4000 c/s, (10) 8000 c/s, (11) 16,000 c/s. Top curve in each figure $M = 0{\cdot}63$, bottom curve $M = 0{\cdot}5$. ○, $r/D = 60$; +, $r/D = 120$; △, $r/D = 240$; ∨, $r/D = 480$; □ (figure 8) additional points, $M = 0{\cdot}63$, 2000 c/s, with vortex generator.

Only a limited number of experiments were carried out with vortex generators, all at the non-dimensional distance of $r/D = 120$, at angles of 30, 60, 90 and 120°, in all the frequency bands used above, at a Mach number of 0·63 and with the three vortex generators with the teeth set respectively at 15, 30 and 40° against the jet axis. As far as could be judged from the results, the directional distribution of noise intensity per octave band remained more or less as without vortex generators: this is illustrated by the 'additional points' in figure 8. However, the reduction in intensity varied with frequency, being a 4 db reduction at low frequencies for the 15° teeth (which were most efficient in noise suppression) and a slight (1–2 db) *increase* at the upper end of the frequency range investigated. These results are generally in accord with previously reported data.

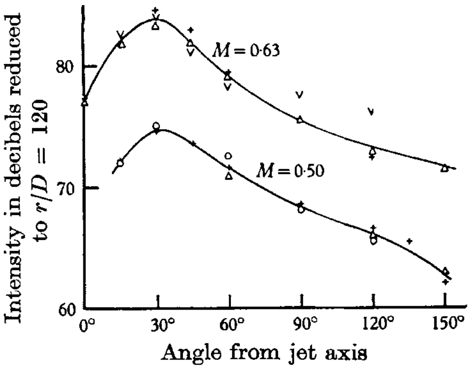

FIGURE 9

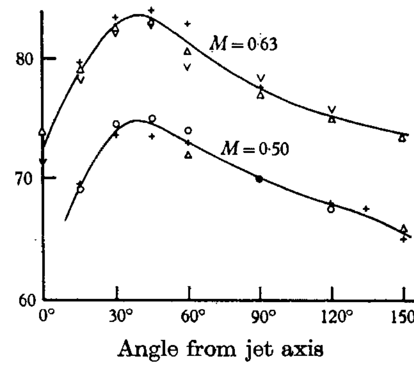

FIGURE 10

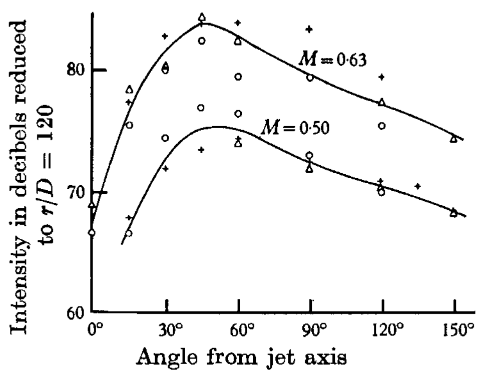

FIGURE 11

FIGURES 9–11. For legend see facing page.

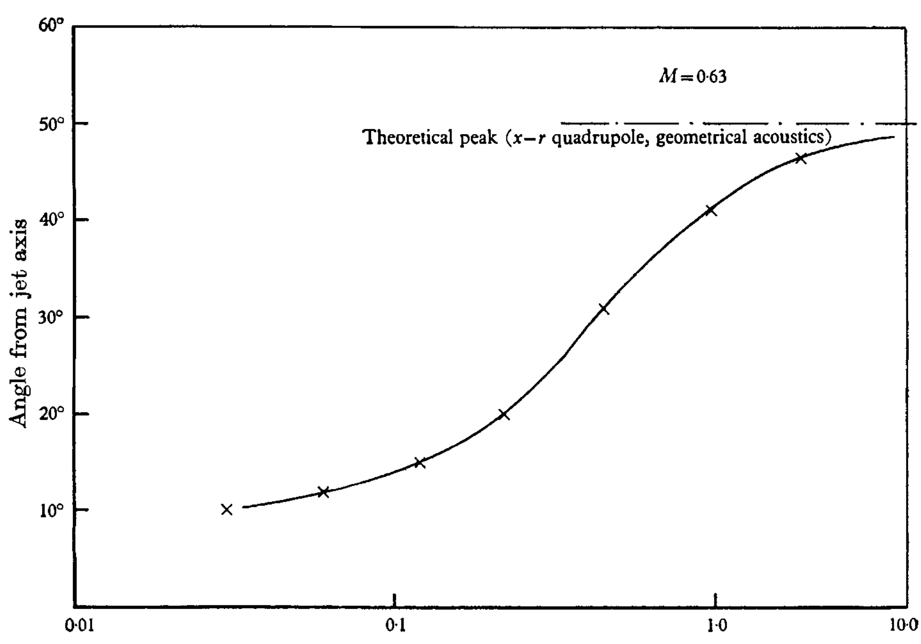

FIGURE 12. Variation of the location of the intensity peak with frequency.

The experimental data may be replotted to show spectra at fixed angular positions. Such curves have often been published in the past and the present results do not differ from them and are therefore not shown. They may be seen, however, in Krishnappa (1967).

Discussion

It may be convenient to begin this discussion with a catalogue of quadrupoles which theory suggests may be important noise radiators.

'Self-noise' quadrupoles radiate noise in all directions and as Proudman (1953) has shown this is likely to be of much the same intensity at all angles θ, except for the convection-amplification effect. Direct measurements of fourth-order velocity correlations in the mixing layer (Jones 1968) tend to confirm this expectation. Thus there is good reason to believe that at low frequencies the self-noise intensity is distributed according to the 'directivity factor'

$$f(\theta) = (1 - M_c \cos\theta)^{-5}. \tag{1}$$

At high frequencies, on the other hand, sound rays emitted parallel to the jet axis are refracted by the non-uniform mean velocity field of the jet, so that a 'zone of silence' is formed directly downstream. At the boundary of this zone geometrical acoustics predicts a sharp intensity peak, located at

$$\theta_p = \cos^{-1}(1+M)^{-1}, \tag{2}$$

where M is the mean velocity Mach number in the emission zone (this is often assumed to be equal to the convection Mach number, M_c; any differences can certainly only be minor). Sound rays emitted at acute angles to the jet axis emerge into the far field at an angle greater than θ_p. The net effect is that high-frequency noise, which at the source may be assumed to be radiated equally in all directions, should show an intensity peak at θ_p given by (2) with a zone of silence at smaller angles.

The 'shear noise' quadrupoles arise from a source term (Csanady 1966)

$$Q = 2\frac{\partial V_i}{\partial x_j}\frac{\partial(\rho v_j')}{\partial x_i}, \tag{3}$$

where the V_i are mean velocity components, v_j' turbulent fluctuations. In the source region of a mixing layer, to a good approximation, we may write

$$V_1 = U(x_2, x_3), \quad V_2 = V_3 = 0.$$

Introducing polar co-ordinates and making use of axial symmetry we have then

$$\left.\begin{aligned} \frac{\partial U}{\partial r} &= \frac{\partial U}{\partial x_2}\cos\phi + \frac{\partial U}{\partial x_3}\sin\phi, \\ \frac{\partial U}{r\,\partial\phi} &= -\frac{\partial U}{\partial x_2}\sin\phi + \frac{\partial U}{\partial x_3}\cos\phi = 0, \end{aligned}\right\} \tag{4}$$

from which

$$\frac{\partial U}{\partial x_2} = \frac{\partial U}{\partial r}\cos\phi, \quad \frac{\partial U}{\partial x_3} = \frac{\partial U}{\partial r}\sin\phi. \tag{5}$$

Applying the same transformation to fluctuating velocity components, carrying out the reduction of the apparent dipole in (3) to quadrupoles (as has been done in Csanady (1966); the details are written out in Krishnappa (1967)) and introducing the convection amplification factor as amended by Jones (1968), we arrive at two quadrupoles only:

(i) an 'x–x' quadrupole radiating as

$$f(\theta) = \frac{\cos^4\theta}{(1-M_c\cos\theta)^3} \tag{6}$$

with intensity proportional to

$$\left(\frac{\partial U}{\partial r}\right)^2 \overline{\left\{\frac{\partial(v_1' v_r')}{\partial t}\right\}^2},$$

and

(ii) an 'x–r' quadrupole radiating as

$$f(\theta) = \frac{\sin^2\theta\cos^2\theta}{(1-M_c\cos\theta)^3} \tag{7}$$

with intensity proportional to

$$\left(\frac{\partial U}{\partial r}\right)^2 \overline{\left\{\frac{\partial}{\partial t}\left[v_r'^2 - \frac{p}{\rho}\right]\right\}^2}.$$

The above equations, (6) and (7), are valid at low frequencies. At high frequencies the x–x quadrupole should again show a sharp peak at the angle given by (2), while the peak of the x–r quadrupole would be at a slightly higher angle which may be calculated from the formulae given in Csanady (1966).

In physical terms, the x–x shear quadrupole arises from fluctuations in the Reynolds shear stress, the x–r quadrupole from those in the radial Reynolds normal stress *plus* the static pressure.

In the light of the above it is now instructive to return to our experimental data, at first on total noise intensity shown in figure 4. The peak is closer to 45° than to any other quadrupole-axis and we may at first look at a comparison with the directional distribution of the 45° shear noise quadrupole, as given by (7), with a realistically chosen convection velocity. Near 45° the theoretical (broken) curve represents the trend of the data correctly (the absolute value having been fitted to the data at one point). Moreover, at 135° (where the x–r quadrupole should again be dominant) the agreement is again almost within the experimental error, suggesting, however, a slightly greater emphasis on forward emission than would follow from a shear-noise only model (if we take the mean of the two curves, $M = 0{\cdot}63$ and $M = 0{\cdot}5$). The accuracy of the data is insufficient to make a reliable quantitative estimate of the proportion of the noise that should be 'self-noise' (with its greater emphasis on forward emission) but the order of magnitude seems to be 80 % shear noise, 20 % self-noise.

Proceeding now to the octave band directional distributions we notice at low frequencies a peak very close to the jet axis. Such a distribution could be produced by the sum of an x–x and an x–r quadrupole, if the intensity of the latter is slightly greater than twice the intensity of the former. When the ratio of the two quadrupoles changes further in favour of the x–r quadrupole, the peak shifts further out, to larger θ. This appears to occur at somewhat higher frequencies (which are, however, still probably too low for the refraction effect to be significant). Since the x–x and x–r shear noise quadrupoles differ mainly in the presence of the pressure fluctuation term in x–r, we may tentatively conclude that with increasing frequency this term becomes dominant.

At very high frequencies the geometrical acoustics approximation should be approached; indeed the peak angle shifts well out but is asymptotically (see figure 12) *not* θ_p given by (2) but something significantly larger. If the x–r shear quadrupole is still dominant at these frequencies, the asymptotic maximum at $M = 0{\cdot}63$ should be about 50°, which appears to be approached by the data points. This suggests that the pressure fluctuation x–r quadrupole is important at all frequencies and dominant at most. It is, in particular, probably responsible for the main lobe of high-intensity noise at angles around 30°, a conclusion already arrived at by Lighthill.

From figure 12 we also conclude that the geometrical acoustics approximation becomes reasonable when the product Strouhal number × Mach number $= fD/a$ exceeds unity (this product is proportional to the ratio of jet diameter to sound wavelength, which is the relevant parameter for deciding whether sound is of 'high' or 'low' frequency). As pointed out above, the change in the position of the peak is probably not alone due to refraction effects, but is at least at lower Strouhal numbers caused by a shifting dominance of quadrupoles.

In regard to the effects of vortex generators we may tentatively conclude that they do not significantly modify the composition of the noise (because the directional distribution remains more or less unaffected), but cause a more or less uniform reduction at lower frequencies, while they contribute their own noise at higher frequencies.

A comparison of the data presented here with those available in the literature (Howes 1960; Mollö-Christensen *et al.* 1964) shows general agreement, but also some minor differences. Mollö-Christensen has stressed the absence of a second peak in the quadrant $90° \leqslant \theta \leqslant 180°$, in contrast to some other previous experiments. The present data agree with Mollö-Christensen's, but it is at least very likely that genuine variations may be caused between jets of different sizes and operating at different Mach numbers by changes in the composition of noise, specifically in the proportionate contribution of the x–r quadrupole.

One of the most difficult aspects of the present experiments, from the point of view of a theoretical explanation, is the lack of a 'dip' around 90°. Any combination of x–x and x–r quadrupoles (shear noise) should produce such a dip, and a well-marked one. This difficulty does not arise if we assume that a large proportion of the noise is 'self-noise', but then the differences between 45° and 135° should be substantially greater than observed because of the more drastic convection-amplification factor $(1-M_c\cos\theta)^{-5}$. There is a possibility that gradients of the *radial* mean velocity also produce shear noise (which would be concentrated around 90°) but it is a little difficult to believe that the order of magnitude of this would be comparable to the noise intensity radiated by the x–x and x–r quadrupoles, because the radial velocities are an order of magnitude smaller than the axial ones. This difficulty remains unresolved.

We may summarize our main conclusions from this study briefly as follows: (i) most of the noise of jets is 'shear noise', as is already suggested (at least by implication) in Jones (1968); (ii) the dominant quadrupole is the x–r, pressure-fluctuation shear-noise quadrupole, as already suggested by Lighthill (1954); (iii) vortex generators do not radically modify the composition of noise; (iv) refraction effects are highly significant only above Strouhal number × Mach number of about unity.

In view of the complexity of the total phenomenon and the difficulty of accurate measurements none of the above conclusions can be regarded as firmly established, but the evidence is suggestive, and alternative explanations (e.g. other specific combinations of shear noise and self-noise quadrupoles) appear to be less self-consistent and more complicated.

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FIGURE 1. Installed nozzle and settling chamber.

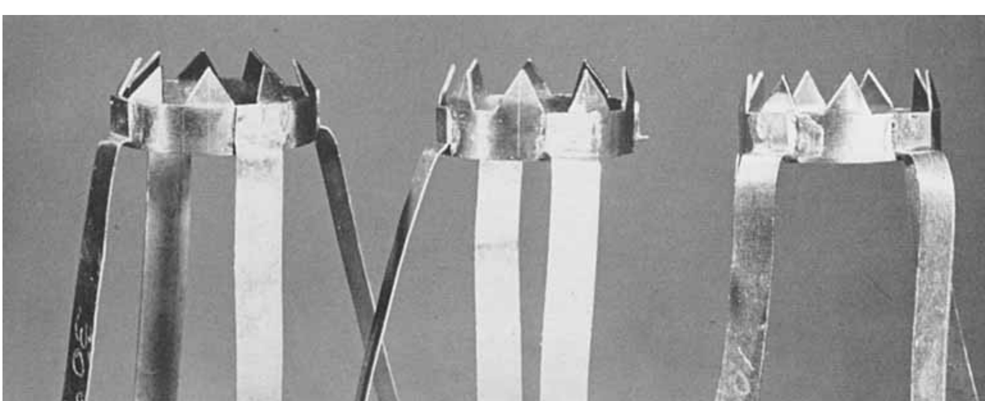

FIGURE 2. Vortex generators used at nozzle exit.